Design Formulas for a Quasi-Optical Diplexer or Multiplexer

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***Abstract*—An equivalent network for a multiple screen bandpass filter with an obliquely incident plane wave has been developed. Based upon this network, the design formulation for this type of filter is derived. Such quasi-optical filter may be used as a diplexer or a multiplexer.**

I. Introduction

AT MILLIMETER and submillimeter wavelengths and in the far infrared region, it would be more appropriate to have frequency diplexing or multiplexing performed by the quasi-optical filters in the open space rather than in waveguide. Because of the large cross section of the quasi-optical filter, lower loss and higher power handling capability would result.

A typical space multiplexer, as shown in Fig. 1, would involve quasi-optical filters with waves incident at an oblique angle. Typical filters of this type consist of one or two layers of resonant grids [1]–[4]. Since the bandpass response for a pair of resonant grids has limited sharpness of skirt, the resultant multiplexer has narrow useful bandwidth and, as a consequence, requires large channel separation. In order to enlarge the useful bandwidth and minimize the required channel separation for a space multiplexer, a more sophisticated channel filter would be required.

A quasi-optical bandpass filter containing multiple layers of screens was developed by Saleh [5], [6]. It may be designed to have the desired number of transmission poles, and consequently to result in the required bandpass response. However, Saleh's development was limited to filters with normally incident plane wave. This paper extends the design formulation for that multiple screen filter for obliquely incident waves.

The local coordinate concept [7], [8] has been used as a basis for deriving the equivalent network of the filter. This network is then reduced to a model of a conventional filter network, from which the filter parameters are derived.

II. Network Representation for a Screen Filter

Consider a multiple screen filter with $N+2$ screens, as shown in Fig. 2, where all the screens are perpendicular to the z axis and each screen is oriented at a particular angle, ϕ_i with respect to the x axis. The incident plane wave is

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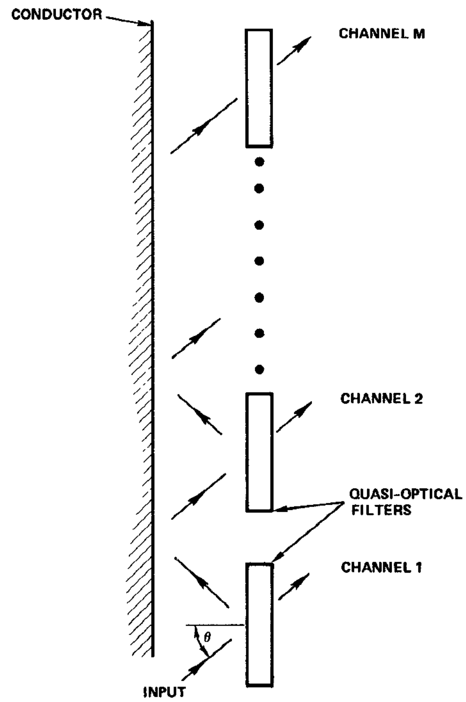


Fig. 1. A multiplexer with quasi-optical bandpass filters.

propagating in the direction of vector $\boldsymbol{k}$, which has its components described as follows:

$$\left.\begin{aligned} k_x &= k\sin\theta\cos\phi \\ k_y &= k\sin\theta\sin\phi \\ k_z &= k\cos\theta \end{aligned}\right\} \quad (1)$$

where θ and ϕ are the polar coordinate angles. Since free space may be considered as a uniform waveguide with infinite cross section, a plane wave with wavenumber $\boldsymbol{k}$ may be decomposed into E-type and H-type modes in this waveguide with a modal propagation direction $\boldsymbol{z}_0$. The E-type (or H-type) mode is defined as a plane wave with $H_y=0$ (or $E_y=0$), propagation constant k_z and transverse wavenumbers k_x and k_y. The total transverse field for this plane wave may, therefore, be represented by the E- and H-type mode as follows:

$$\left.\begin{aligned} \boldsymbol{E}_t(x,y,z) &= V'(z)\boldsymbol{e}'(x,y) + V''(z)\boldsymbol{e}''(x,y) \\ \boldsymbol{H}_t(x,y,z) &= I'(z)\boldsymbol{h}'(x,y) + I''(z)\boldsymbol{h}''(x,y) \end{aligned}\right\} \quad (2)$$

where $\boldsymbol{e}(x,y)$ and $\boldsymbol{h}(x,y)$ are the mode functions, $V(z)$ and $I(z)$ are the mode amplitudes, the prime denotes E-type mode, and double prime denotes H-type mode. The mode

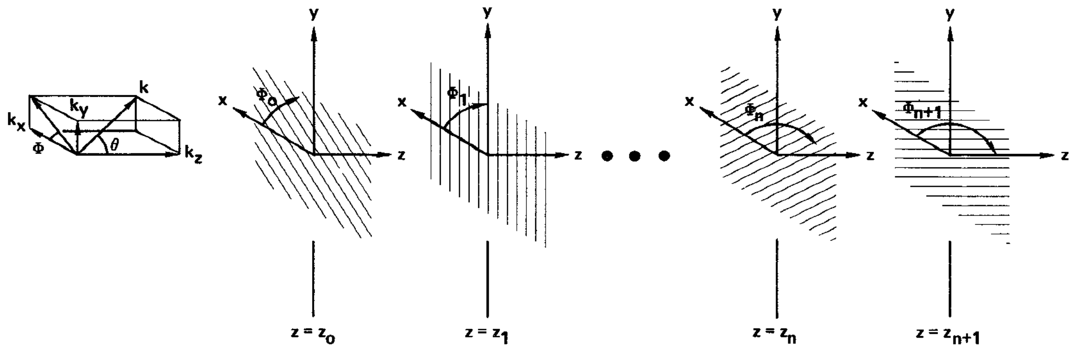


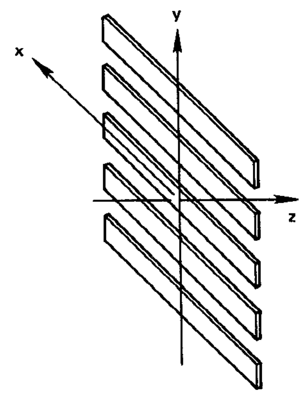
Fig. 2. A multiple screen filter with the obliquely incident plane wave.

functions, characteristic impedances, and propagation constants for the E- and H-type modes are

$$\left.\begin{aligned} e'(x,y)&=\frac{1}{2\pi}\left[x_0\left(-\frac{k_xk_y}{k^2-k_y^2}\right)+y_0\right]e^{-j(k_xx+k_yy)}\\ h'(x,y)&=\frac{1}{2\pi}\left[-x_0\right]e^{-j(k_xx+k_yy)}, \qquad -\infty<\frac{k_x}{k_y}<\infty\\ Z_0'&=\frac{k^2-k_y^2}{k_z\omega\epsilon}\end{aligned}\right\} \quad (3a)$$

$$\left.\begin{aligned} e''(x,y)&=\frac{1}{2\pi}\left[x_0\right]e^{-j(k_xx+k_yy)}\\ h''(x,y)&=\frac{1}{2\pi}\left[x_0\left(-\frac{k_xk_y}{k^2-k^2}\right)+y_0\right]e^{-j(k_xx+k_yy)}\\ Z_0''&=\frac{k_z\omega\mu}{k^2-k_y^2}, \qquad -\infty<\frac{k_x}{k_y}<\infty.\end{aligned}\right\} \quad (3b)$$

The decomposition of a plane wave into E- and H-type modes in an appropriate coordinate system would result in a simplified network representation for a parallel strip grating. For example, a simple capacitive loading in the E-type modal transmission line and a simple inductive loading in the H-type transmission line, as shown in Fig. 3, may be used to represent a parallel strip grating which is oriented in the x direction. This simple network representation may be applied to every screen if the local coordinate system for each screen were selected in such a way that the x axis is always aligned with the screen orientation. With this network as an element, the equivalent network for the complete screen filter is derived in Fig. 4. It is a cascade network, which contains three types of elemental networks, the loading network L, the coupling network R, and the uncoupled transmission lines. The loading network L is the representation of the grating in its local coordinate system; the uncoupled transmission lines represent the separation between two adjacent screens; and the coupling network R relates the modal amplitudes for E- and H-type modes between two adjacent coordinate systems, and may be derived by coordinate rotation.



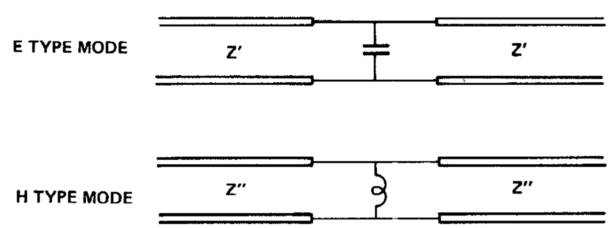


Fig. 3. Equivalent network for a parallel strip grating.

The total transverse field of a plane wave with a fixed wave vector $\boldsymbol{k}$ may be represented by a summation of E- and H-type modes either in coordinate 1 or in coordinate 2. Thus

$$\left.\begin{aligned} E_t(x,y,z)&=V_1'(z_1)e'(x_1,y_1)+V_1''(z_1)e''(x_1,y_1)\\ &=V_2'(z_2)e'(x_2,y_2)+V_2''(z_2)e''(x_2,y_2)\\ H_t(x,y,z)&=I_1'(z_1)h'(x_1,y_1)+I_1''(z_1)h''(x_1,y_1)\\ &=I_2'(z_2)h'(x_2,y_2)+I_2''(z_2)h''(x_2,y_2)\end{aligned}\right\} \quad (4)$$

where the subscripts 1 and 2 refer to the coordinates 1 and 2, respectively. The mode functions for the E- and H-type modes are as derived in (3), provided that the unit vectors and the coordinate variables are referred to the individual coordinate systems.

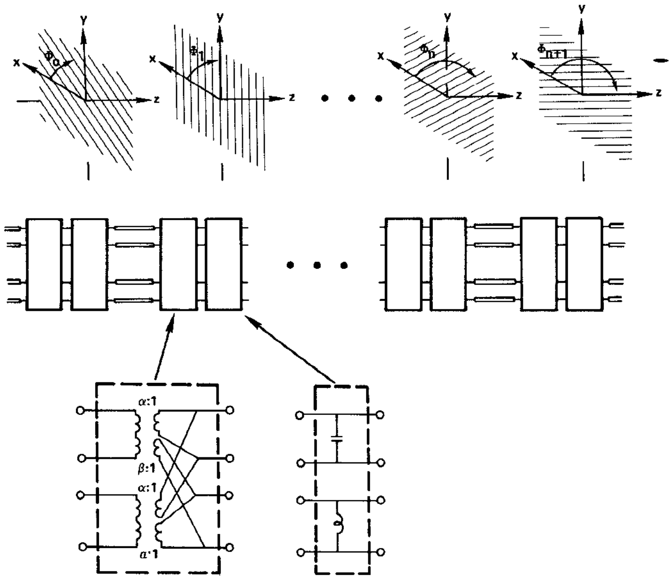


Fig. 4. Equivalent network for a multiple screen filter with $N+2$ screens.

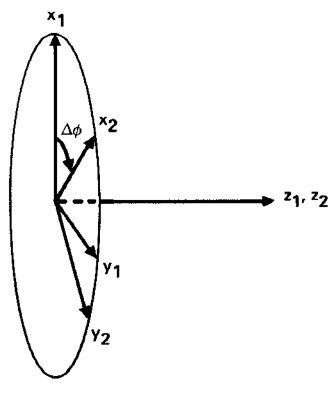


Fig. 5. Coordinate relationship for Σ_1 and Σ_2.

Consider coordinate system 2, which is generated from coordinate system 1 by a rotation of $\Delta\phi$ about the z axis (see Fig. 5). Then the coordinate relationships are as follows:

$$\begin{bmatrix} x_{01} \\ y_{01} \\ z_{01} \end{bmatrix} = \tau \begin{bmatrix} x_{02} \\ y_{02} \\ z_{02} \end{bmatrix} \tag{5}$$

$$\begin{bmatrix} k_{x1} \\ k_{y1} \\ k_{z1} \end{bmatrix} = \tau \begin{bmatrix} k_{x2} \\ k_{y2} \\ k_{z2} \end{bmatrix} \tag{6}$$

$$\begin{bmatrix} x_{1} \\ y_{1} \\ z_{1} \end{bmatrix} = \tau \begin{bmatrix} x_{2} \\ y_{2} \\ z_{2} \end{bmatrix} \tag{7}$$

where

$$\tau = \begin{bmatrix} \cos\Delta\phi & \sin\Delta\phi & 0 \\ -\sin\Delta\phi & \cos\Delta\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{8}$$

Utilizing the orthogonality of the vector mode functions, (4) may be rewritten in matrix form as follows:

$$\begin{bmatrix} V_1'(z) \\ I_1'(z) \\ V_1''(z) \\ I_1''(z) \end{bmatrix} = R(\theta, \phi_1 - \phi_2) \begin{bmatrix} V_2'(z) \\ I_2'(z) \\ V_2''(z) \\ I_2''(z) \end{bmatrix} \tag{9}$$

with the relationship in (6)–(8), $R(\theta, \phi_1 - \phi_2)$ is found to be

$$R(\theta, \phi_1 - \phi_2) = \begin{bmatrix} \alpha & 0 & \delta & 0 \\ 0 & \beta & 0 & \gamma \\ -\gamma & 0 & \beta & 0 \\ 0 & -\delta & 0 & \alpha \end{bmatrix} \tag{10}$$

where

$$\alpha = \int_s e'(x_2,y_2) \times h'^*(x_1,y_1) \cdot z_0 \, ds = \frac{\cos(\phi_1 - \phi_2) - \sin^2\theta \sin\phi_1 \sin\phi_2}{1 - \sin^2\theta \sin^2\phi_2}$$

$$\beta = \int_s h'(x_2,y_2) \times e'^*(x_1,y_1) \cdot z_0 \, ds = \frac{\cos(\phi_1 - \phi_2) - \sin^2\theta \sin\phi_1 \sin\phi_2}{1 - \sin^2\theta \sin^2\phi_1}$$

$$\gamma = \int_s h''(x_2,y_2) \times e'^*(x_1,y_1) \cdot z_0 \, ds = \frac{\sin(\phi_1 - \phi_2)\cos^2\theta}{(1 - \sin^2\theta \sin^2\phi_1)(1 - \sin^2\theta \sin^2\phi_2)}$$

$$\delta = \int_s e''(x_2,y_2) \times h'^*(x_1,y_1) \cdot z_0 \, ds = \sin(\phi_1 - \phi_2). \tag{11}$$

The inductive loading approaches a short-circuit and the capacitive loading approaches an open-circuit when the grating period p approaches zero. This is a low-frequency approximation, which is a good representation for a very fine screen. In this limit, the network in Fig. 4(b) is simplified to the network in Fig. 6(a). A section of H-type modal transmission line with a short-circuit at one end and a coupling transformer at the other end may be equivalently represented by a susceptance. Therefore, the network in Fig. 6(a) is further reduced to the network in Fig. 6(b).

III. Design Formulation for the Filter

The network in Fig. 6(b) is a cascade network with $N+1$ cells which represents a multiple screen filter of $N+2$ screens. Each cell contains a section of quarter-wavelength transmission line, a transformer, and a shunt loading. In order to apply conventional filter design techniques in the design of the desired screen filter, this network must be modified by some approximation to a circuit with admittance inverters and shunt inductances.

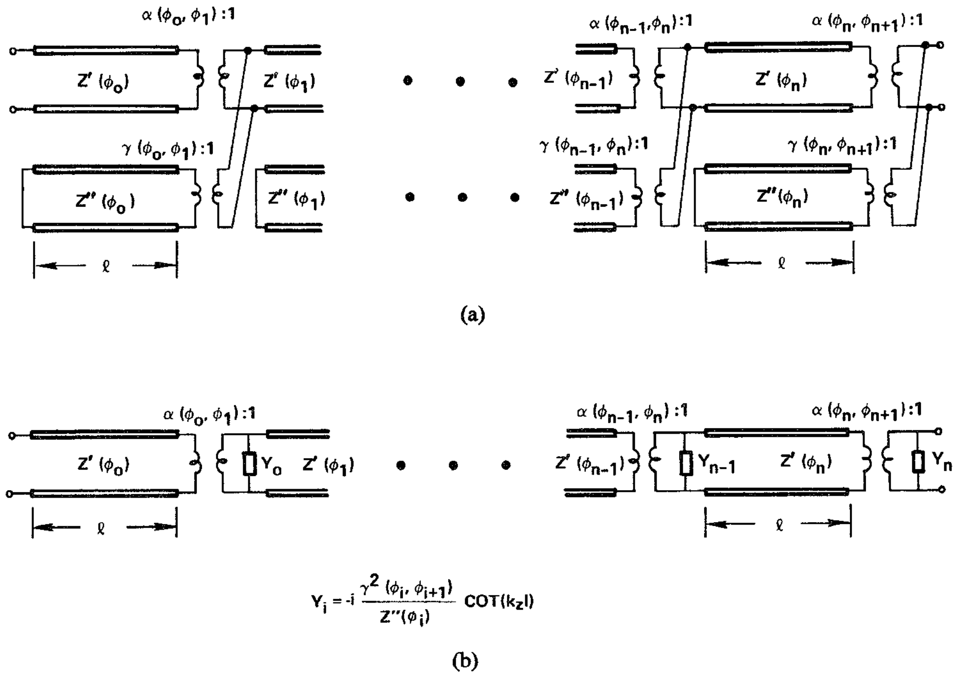


Fig. 6. Simplified equivalent networks for a multiple screen filter.

The network behavior for a section of transmission line in the neighborhood of quarter-wavelength is equivalent to an admittance inverter and two shunt inductances in cascade. Therefore, a unit cell of the equivalent network for the screen filter may be modified as illustrated in Fig. 7 from (a) to (b) and then to (c). The admittance inverter with the cascading transformer, as shown in Fig. 7(c), may be combined into one equivalent admittance inverter. Also, two shunt loadings in this unit cell may be combined with another shunt loading in the next unit cell. Therefore, the overall cascade network for the screen filter is converted to a conventional filter network except for two shunt reactances B_0 and B_{n+1} at the input and output ports (see Fig. 8). B_0 and B_{n+1} may be absorbed into the reference phase shift for the filter. As is shown in Fig. 8, the equivalent admittance inverter $J_{i,i+1}$ and the equivalent shunt inductance B_i may be expressed in terms of the orientations of the individual screens as follows:

$$J_{i,i+1} = \alpha(\phi_i, \phi_{i+1})/Z'(\phi_i) \tag{12}$$

$$B_{i+1} = -\frac{\alpha^2(\phi_i,\phi_{i+1})\cos k_z l}{Z'(\phi_i)} - \frac{\cos k_z l}{Z'(\phi_{i+1})} - \frac{\gamma^2(\phi_i,\phi_{i+1})\cot k_z l}{Z''(\phi_i)}. \tag{13}$$

The slope parameter for B_{i+1} may be obtained from the fact that

$$b_{i+1} = \frac{\omega_0}{2}\frac{dB_{i+1}}{d\omega}\bigg|_{\omega=\omega_0} = \frac{\pi}{4}\left\{\frac{\alpha^2(\phi_i,\phi_{i+1})}{Z'(\phi_i)} + \frac{1}{Z'(\phi_{i+1})} + \frac{\gamma^2(\phi_i,\phi_{i+1})}{Z''(\phi_i)}\right\}. \tag{14}$$

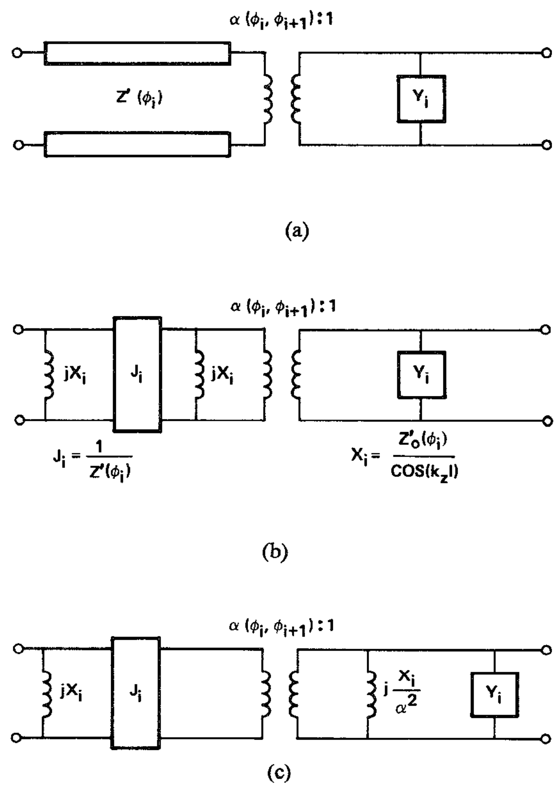


Fig. 7. Equivalent representation for a unit cell of the network in Fig. 3(b).

According to the conventional design method for an N-pole Chebyshev or maximally flat filter, the admittance inverter and the slope parameters for the shunt loading should be related to the element values of the filter as

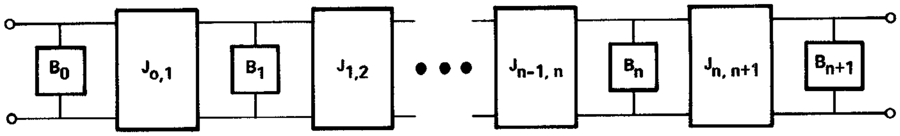


Fig. 8. Equivalent filter network for a multiple screen filter with $N+2$ screens.

TABLE I
SCREEN ORIENTATIONS FOR A FIVE-SCREEN (THREE-POLE) CHEBYSHEV FILTER WITH 0.01-dB RIPPLE AND 1-PERCENT BANDWIDTH

θ	Φ_1	$\Phi_2-\Phi_1$	$\Phi_3-\Phi_2$	$\Phi_4-\Phi_3$
0.00000	80.90826	88.84792	-88.84792	-80.90826
15.00000	81.21314	88.18973	-88.18973	-81.21314
30.00000	82.21314	86.10312	-86.10312	-82.10982
45.00000	83.54412	82.11946	-82.11946	-83.54412
60.00000	85.42533	74.70299	-74.70299	-85.42533

TABLE II
SCREEN ORIENTATION FOR A SEVEN-SCREEN (FIVE-POLE) CHEBYSHEV FILTER WITH 0.01-dB RIPPLE AND 1-PERCENT BANDWIDTH

θ	Φ_1	$\Phi_2-\Phi_1$	$\Phi_3-\Phi_2$	$\Phi_4-\Phi_3$	$\Phi_5-\Phi_4$	$\Phi_6-\Phi_5$
0.00000	81.71389	89.09401	89.37266	-89.37266	-89.09401	-81.71389
15.00000	81.92251	88.49695	90.01949	-90.01949	-88.49695	-81.99251
30.00000	82.81153	86.60439	92.05940	-92.05940	-86.60439	-82.81153
45.00000	84.12037	82.98932	95.91282	-95.91282	-82.98932	-84.12037
60.00000	85.83517	76.23085	102.98794	-102.98794	-76.23085	-85.83517

TABLE III
SCREEN ORIENTATION FOR A NINE-SCREEN (SEVEN-POLE) CHEBYSHEV FILTER WITH 0.01-dB RIPPLE AND 1-PERCENT BANDWIDTH

θ	Φ_1	$\Phi_2-\Phi_1$	$\Phi_3-\Phi_2$	$\Phi_4-\Phi_3$	$\Phi_5-\Phi_4$	$\Phi_6-\Phi_5$	$\Phi_7-\Phi_6$	$\Phi_8-\Phi_7$
0.00000	81.92918	89.14557	89.42312	89.46733	-89.46733	-89.42312	-89.14557	-81.92918
15.00000	82.20074	88.56449	90.05080	88.80274	-88.80274	-90.05080	-88.56449	-82.20074
30.00000	82.99892	86.72261	92.03055	96.69974	-86.69974	-92.03055	-86.72261	-82.99892
45.00000	84.27415	83.20379	95.77215	82.70019	-92.70019	-95.77215	-83.20379	-84.27415
60.00000	85.94447	76.61830	102.65327	75.29475	-75.29475	-102.65327	-76.61830	-85.94447

follows:

$$\frac{J_{0,1}}{\sqrt{b_1}} = \sqrt{\frac{G_A BW}{g_0 g_1}} \tag{15a}$$

$$\frac{J_{i,i+1}}{\sqrt{b_i b_{i+1}}} = \frac{BW}{\sqrt{g_i g_{i+1}}}, \qquad i=1,2,\cdots,N-1 \tag{15b}$$

$$\frac{J_{n,n+1}}{\sqrt{b_n}} = \sqrt{\frac{G_B BW}{g_n g_{n+1}}} \tag{15c}$$

where BW is the fractional bandwidth, $G_A = 1/Z'(\phi_0)$, and $G_B = 1/Z'(\phi_{n+1})$. As is shown in (12) and (14), $J_{0,1}$ and b_1 are both functions of ϕ_0 and ϕ_1. Therefore, (15a) involves ϕ_0 and ϕ_1. Since one can always select an initial coordinate system that makes $\phi_0 = 0$, (15a) is dependent upon ϕ_1 only. Thus ϕ_1 is determined. With $\phi_0 = 0$ and ϕ_i known, ϕ_{i+1} may be obtained from (15b). Applying this procedure, $\phi_2, \phi_3, \phi_4, \cdots, \phi_n$ may be obtained from 15(b) as i is incremented from 2 to $n-1$. Finally, (15c) is solved for ϕ_{n+1}.

As a demonstration of this method, three-pole, five-pole, and seven-pole Chebyshev filters have been designed and their parameters are tabulated in Tables I, II, and III. All filters are designed to have 0.01-dB ripple and 1-percent bandwidth.

IV. CONCLUSION

The design formulation for a multiple screen filter with a plane wave incident at an arbitrary angle has been presented. Thereby, a multiple screen filter of this type may be designed to have any desired skirt sharpness and bandwidth as would be a conventional filter. For example, a seven-screen filter with the screen orientations given in Table II would have the typical transmission response of a five-pole Chebyshev filter (see Fig. 9). As a result, a

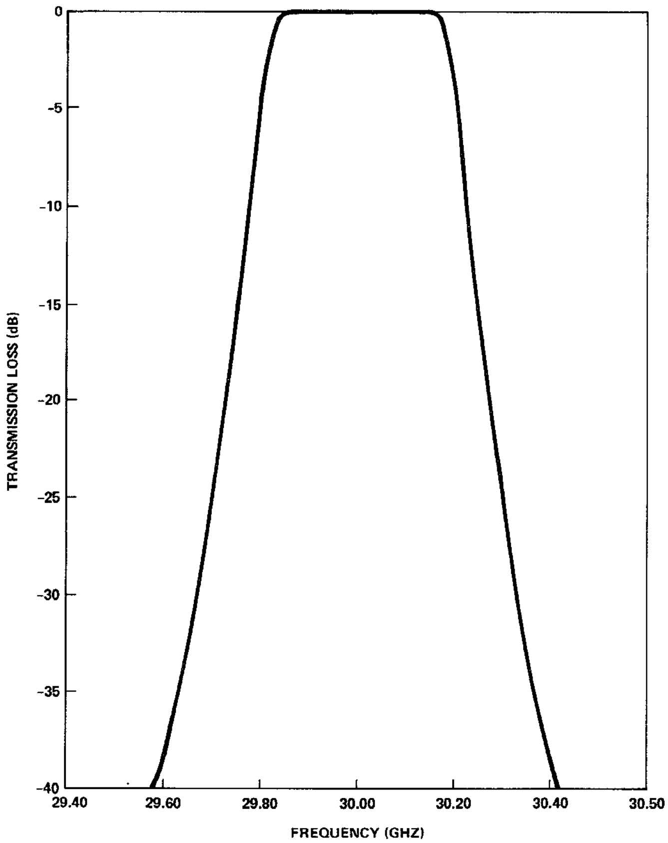


Fig. 9. The transmission response for a seven-screen filter with the plane wave incident at 45°. The screen orientations are given in Table II.

quasi-optical multiplexer with desired bandwidth and channel separation may be realized.

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